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The string effective action in the dual formulation of $D=10$ supergravity

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We construct the quartic effective action for the heterotic string using the dual formulation of $d=10$, $N=1$ supergravity. We find that the symmetry between the Yang–Mills and gravity sectors which was present in the two-form formulation, also occurs when the six-index gauge field is used.

The low-energy effective action of the heterotic string is a useful link between the world of elementary particle physics and strings. This effective action describes a standard supergravity theory, modified by higher-derivative terms.

Recently we constructed [1] all terms in the effective action up to and including quartic terms in the Riemann tensor, using the component formulation of $d=10$, $N=1$ supergravity coupled to Yang–Mills theory [2]. In this construction, we employed a useful analogy between the $d=10$, $N=1$ supersymmetric Yang–Mills multiplet, and a multiplet of fields consisting of the gauge fields of supergravity itself, including in particular the $SO(9, 1)$ spin-connection. In this Letter we extend this method to the dual formulation of $d=10$, $N=1$ supergravity [3], and construct the supersymmetric quartic effective action and the corresponding supersymmetry transformation rules for this dual version^{#1}. For convenience, we summarize the formulae that are needed for an analysis of compactification to four dimensions at the end of the paper (eqs. (21), (22)).

There are several reasons to be interested in such an extension. In the first place, the dual formulation is an equivalent formulation of the supergravity theory, which, when suitable modifications are included, is equally capable of achieving the anomaly cancellations [5]. Secondly, it was shown recently that the effective action of the heterotic string admits five-branes as classical solutions [6]. The natural background to which these five-branes couple is the dual formulation of ten-dimensional supergravity. Thirdly, the dual formulation is much closer to $D=10$ conformal supergravity [7], and therefore to possible off-shell formulations.

Our aim, then, is to generalize the results of ref. [1] to the dual formulation of $d=10$, $N=1$ supergravity. We start therefore by presenting the action and supersymmetry transformation rules in this formulation. These can be derived from the two-form formulation by a standard duality transformation. The result is^{#2}

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^{#1} For earlier results in the dual formulation obtained by superspace methods, see ref. [4] and references therein.

^{#2} We use throughout this paper the conventions of ref. [1].

$$\begin{aligned} \mathcal{L}_0(R) = & e\phi^{-3} \left[-\frac{1}{2} R(\omega(e)) + \frac{1}{4} H_{abc} H^{abc} + \frac{9}{2} (\phi^{-1} \partial_\mu \phi)^2 - \frac{1}{2} \bar{\psi}_\mu \Gamma^{\mu\nu\rho} \mathcal{L}_\nu(\omega(e)) \psi_\rho + 2\sqrt{2} \bar{\lambda} \Gamma^{\mu\nu} \mathcal{L}_\mu(\omega(e)) \psi_\nu \right. \\ & + 4\bar{\lambda} \mathcal{L}(\omega(e)) \lambda + 3\sqrt{2} \bar{\psi}_\mu \Gamma^\nu \Gamma^\mu \lambda (\phi^{-1} \partial_\nu \phi) - \frac{3}{2} \bar{\psi}_\mu \Gamma^\mu \psi_\nu (\phi^{-1} \partial^\nu \phi) \\ & \left. + \frac{1}{16} \sqrt{2} H^{abc} (\bar{\psi}_\mu \Gamma^{[\mu} \Gamma_{abc} \Gamma^{\nu]} \psi_\nu + 4\sqrt{2} \bar{\psi}_\mu \Gamma^\mu_{abc} \lambda - 8\bar{\lambda} \Gamma_{abc} \lambda) \right], \end{aligned} \quad (1)$$

$$\begin{aligned} \mathcal{L}_\beta(F^2) = & e\phi^{-3} \beta \text{tr} \left[-\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \frac{1}{2} \bar{\chi} \mathcal{L}(\omega(e, \psi), A) \chi - \frac{1}{4} \bar{\chi} \Gamma^\mu F^{ab} F_{ab} (\psi_\mu + \frac{1}{3} \sqrt{2} \Gamma_\mu \lambda) - \frac{1}{16} \sqrt{2} \bar{\chi} \Gamma^{abc} \chi H_{abc} \right] \\ & - \frac{1}{4} i \beta \sqrt{2} \epsilon^{\mu_1 \dots \mu_6 \nu_1 \dots \nu_4} A_{\mu_1 \dots \mu_6} \text{tr} F_{\nu_1 \nu_2} F_{\nu_3 \nu_4}. \end{aligned} \quad (2)$$

The action (1) + (2) is invariant under the following supersymmetry transformations:

$$\begin{aligned} \delta_0 e_\mu^a = & \frac{1}{2} \bar{\epsilon} \Gamma^a \psi_\mu, \quad \delta_0 \psi_\mu = (\partial_\mu - \frac{1}{4} \Omega_{\mu+}^{ab} \Gamma_{ab}) \epsilon, \quad \delta_0 A_{\mu_1 \dots \mu_6} = \frac{3}{4 \times 6!} \sqrt{2} \phi^{-3} \bar{\epsilon} \Gamma_{\mu_1 \dots \mu_5} (\psi_{\mu_6} + \frac{1}{3} \sqrt{2} \Gamma_{\mu_6} \lambda), \\ \delta_0 \lambda = & -\frac{1}{8} \sqrt{2} \phi^{-1} \mathbb{P} \phi \epsilon + \frac{1}{8} \Gamma^{abc} \epsilon H_{abc}, \quad \phi^{-1} \delta_0 \phi = -\frac{1}{3} \sqrt{2} \bar{\epsilon} \lambda, \quad \delta_0 A_\mu = \frac{1}{2} \bar{\epsilon} \Gamma_\mu \chi, \quad \delta_0 \chi = -\frac{1}{4} \Gamma^{ab} \epsilon F_{ab}. \end{aligned} \quad (3)$$

In these actions we have not written the four-fermion interactions, nor do we discuss contributions to the fermion transformation rules which are bilinear in fermions. In an accompanying paper [8], in which we discuss the duality transformations between effective string actions, these terms are shown explicitly. Here we keep only those contributions which are relevant for compactification to four dimensions.

In this paper we use subscripts $\alpha^n(\beta^n)$ to indicate the order in α (the inverse string tension) and β (the inverse Yang–Mills coupling squared) of actions and transformation rules (\mathcal{L}_0 is of $O(\alpha^0, \beta^0)$).

In (1)–(3) we use the symbol H_{abc} for the dual of the field strength of the six-index gauge field, and \hat{H}_{abc} for a supercovariant combination which includes bilinear fermions:

$$H_{abc} = \frac{2}{3} i e^{-1} \phi^3 \epsilon^{\lambda_1 \dots \lambda_7}_{abc} R(A)_{\lambda_1 \dots \lambda_7}, \quad \hat{H}_{abc} = \frac{2}{3} i e^{-1} \phi^3 \epsilon^{\lambda_1 \dots \lambda_7}_{abc} \hat{R}(A)_{\lambda_1 \dots \lambda_7} - \frac{1}{3} \sqrt{2} \bar{\lambda} \Gamma_{abc} \lambda. \quad (4)$$

Furthermore, the torsionful spin-connection $\Omega_{\mu\pm}^{ab}$ is given by

$$\Omega_{\mu\pm}^{ab} = \omega_\mu^{ab}(e, \psi) \pm \frac{3}{2} \sqrt{2} \hat{H}_{abc}. \quad (5)$$

With the use of the expressions (4), the action is quite similar to the formulation with a two-index gauge field. In that case H is the field strength of this gauge field. The difference between the two actions lies in the fact that in the dual formulation there are no Chern–Simons terms in H . Their role is taken over by the last term in (2), which is absent in the two-index version. Also note that there are no $O(\beta)$ modifications to the supersymmetry transformation rules (except of higher order in fermions, which we do not discuss here).

We now proceed with the construction of a supersymmetric action quadratic in the Riemann tensor. We first obtain a combination of the supergravity fields which transforms under supersymmetry as an $SO(9, 1)$ Yang–Mills gauge field. With the corresponding spinor field, it forms an $SO(9, 1)$ Yang–Mills multiplet, so that in constructing an invariant action (2) may be used.

We find again that the appropriate $SO(9, 1)$ gauge field is the combination $\Omega_{\mu-}^{ab}$, and the corresponding fermion the gravitino field strength ψ^{ab} . The transformation rules of $\Omega_{\mu-}^{ab}$ and ψ^{ab} , which follow from (3), are

$$\delta_0 \Omega_{\mu-}^{ab} = \frac{1}{2} \bar{\epsilon} \Gamma_\mu \psi^{ab} + \frac{3}{4} e^{-1} e_{\mu c} \phi^3 \bar{\epsilon} \Gamma^{[ab} \mathcal{L}^{c]} + \frac{1}{8} \sqrt{2} e^{-1} \phi^3 \bar{\epsilon} \Gamma_\mu^{ab} A, \quad (6)$$

$$\delta_0 \psi^{ab} = -\frac{1}{4} F^{cd} \epsilon R_{cd}^{ab} (\Omega_-) - \frac{1}{8 \times 6!} \sqrt{2} \Gamma^{ab \lambda_1 \dots \lambda_6} \epsilon \mathcal{L}_{\lambda_1 \dots \lambda_6}. \quad (7)$$

In these transformation rules the equations of motion of the supergravity fields occur. The complete set of equations of motion (Φ , \mathcal{E}_a^μ , A , Ψ^μ , $\mathcal{A}^{\mu_1 \dots \mu_6}$, \mathcal{A}^μ and \mathcal{X} denote the equations of motion of ϕ , e_μ^a , λ , ψ_μ , $A_{\mu_1 \dots \mu_6}$, A_μ and χ , respectively) is given by

$$\Phi = e\phi^{-3} \left[\frac{3}{2} R(\omega) - 9 \mathcal{L}_a(\omega) (\phi^{-1} \partial_a \phi) + \frac{27}{2} (\phi^{-1} \partial_\mu \phi)^2 + \frac{9}{4} H^{\mu\nu\lambda} H_{\mu\nu\lambda} \right], \quad (8)$$

$$\begin{aligned}
\mathcal{E}_a^\mu &= e\phi^{-3} e^\mu{}_\rho e^{\nu a} [R_\nu{}^h(\omega) - 3\mathcal{I}_\nu(\omega)(\phi^{-1}\partial^h\phi) + \frac{9}{2}H_{\nu\lambda\rho}H^{h\lambda\rho}] - \frac{1}{3}e^\mu{}_a\Phi, \\
\Lambda &= e\phi^{-3} [8\mathcal{P}(\omega)\lambda + \sqrt{2}\Gamma^{\mu\nu}\psi_{\mu\nu} - 12(\phi^{-1}\mathcal{P}\phi)\lambda - \sqrt{2}\Gamma^{abc}\lambda H_{abc}], \\
\mathcal{P}_\mu &= e\phi^{-3} [\Gamma^\nu\psi_{\mu\nu} + 2\sqrt{2}\mathcal{D}_\mu(\Omega_+)\lambda] - \frac{1}{4}\sqrt{2}\Gamma_\mu\Lambda, \\
\mathcal{A}_{\lambda_1\dots\lambda_6} &= 4 \times 7! \mathcal{I}_\mu [e\phi^3 R(A)^\mu{}_{\lambda_1\dots\lambda_6}], \\
\mathcal{A}^\mu &= e^\mu{}_a \mathcal{I}_\nu(\Omega_+, A)(e\phi^{-3}F^{\nu a}), \\
\mathcal{X} &= -e\phi^{-3} [\mathcal{P}(\omega, A)\chi + \frac{1}{2}\sqrt{2}\Gamma^{\mu\nu}F_{\mu\nu}\lambda - \frac{3}{2}(\phi^{-1}\mathcal{P}\phi)\chi + \frac{1}{8}\sqrt{2}\Gamma\cdot H\chi]. \quad (8 \text{ cont'd})
\end{aligned}$$

Note that the transformation rules (6) and (7) are analogous to those of the Yang–Mills multiplet up to equation of motion terms. Such terms do not obstruct the construction of an invariant action, since their contribution to the variation of the total action can always be cancelled by a modification of the supersymmetry transformation rules of the supergravity fields. This feature also occurs in a similar construction of the R^2 -action in $d=4$ on-shell Poincaré supergravity [9].

From the above, it is clear that the quadratic effective action is given by

$$\begin{aligned}
\mathcal{L}_\alpha(R^2) &= e\phi^{-3}\alpha[-\frac{1}{4}R^{\mu\nu ab}(\Omega_-)R_{\mu\nu}{}^{ab}(\Omega_-) - \frac{1}{2}\bar{\psi}^{ab}\mathcal{P}(\omega(e, \psi), \Omega_-)\psi_{ab} - \frac{1}{4}\bar{\psi}^{ab}\Gamma^\mu\Gamma^{cd}R_{cdab}(\Omega_-)(\psi_\mu + \frac{1}{3}\sqrt{2}\Gamma_\mu\lambda) \\
&\quad - \frac{1}{16}\sqrt{2}\bar{\psi}^{ab}\Gamma^{\mu\nu\rho}\psi_{ab}H_{\mu\nu\rho}] - \frac{1}{4}i\alpha\sqrt{2}\epsilon^{\mu_1\dots\mu_6\nu_1\dots\nu_4}A_{\mu_1\dots\mu_6}R_{\nu_1\nu_2}{}^{ab}(\Omega_-)R_{\nu_3\nu_4 ab}(\Omega_-). \quad (9)
\end{aligned}$$

However, the combination $\mathcal{L}_0(R) + \mathcal{L}_\alpha(R^2)$ is not invariant under the combined transformations (3), (6), (7). The variation of the action takes the form

$$\delta(\mathcal{L}_0(R) + \mathcal{L}_\alpha(R^2)) = \alpha[(\delta_{\text{extra}}\Omega_{\mu-}^{ab})\mathcal{A}^{\mu ab} + (\delta_{\text{extra}}\bar{\psi}^{ab})\mathcal{X}^{ab}], \quad (10)$$

where $\mathcal{A}^{\mu ab}$ and \mathcal{X}^{ab} are the Yang–Mills equations of motion in the last two lines of (8), but with the Yang–Mills fields replaced by the $\text{SO}(9, 1)$ -multiplet $\Omega_{\mu-}^{ab}$, $\bar{\psi}^{ab}$. The variations δ_{extra} in (10) are the equation of motion terms in (6), (7). Therefore the variation (10) can be cancelled by additional $\mathcal{O}(\alpha)$ transformation rules of the supergravity fields:

$$\delta_\alpha\psi_\mu = \frac{1}{4}\alpha e^{-1}\phi^3 e_{\mu[c}\Gamma_{ab]}\epsilon\mathcal{A}^{ab c}, \quad \delta_\alpha\lambda = -\frac{1}{6}\sqrt{2}\Gamma^\mu\delta_\alpha\psi_\mu, \quad \delta_\alpha A_{\mu_1\dots\mu_6} = \frac{1}{8 \times 6!}\alpha\sqrt{2}\epsilon\Gamma_{ab\mu_1\dots\mu_6}\mathcal{X}^{ab}. \quad (11)$$

However, there is another way to cancel (10). The reason is that $\mathcal{A}^{\mu ab}$ and \mathcal{X}^{ab} , with the $\text{SO}(9, 1)$ interpretation, can themselves be expressed in terms of the $\mathcal{O}(\alpha^0)$ supergravity equations of motion. This was shown in detail in ref. [1], where the result was presented in the form of a Lemma. Ultimately this Lemma expresses the contents of the Gauss–Bonnet theorem. We now present it in the dual formulation:

Lemma. For arbitrary transformations $\delta\Omega_{\mu-}^{ab}$ and $\delta\bar{\psi}^{ab}$ the variation of $\mathcal{L}_\alpha(R^2)$, given in eq. (9), is given by

$$\begin{aligned}
\delta\mathcal{L} &= \alpha\delta\bar{\psi}^{\lambda\rho}\{2e\phi^{-3}\mathcal{I}_\lambda(\Omega_+)[e^{-1}\phi^3(\Psi_\rho + \frac{1}{4}\sqrt{2}\Gamma_\rho\Lambda)] - \Gamma^a\psi_\lambda(\mathcal{E}_{\rho a} + \frac{1}{3}e_{\rho a}\Phi)\} \\
&\quad - 2\alpha(\mathcal{E}_\rho^a + \frac{1}{3}e_\rho^a\Phi)[e^{-1}\phi^3\mathcal{I}_\lambda(\Omega_+)(e\phi^{-3}e^{\nu a}\delta\Omega_{\nu-}^{\lambda\rho})] \\
&\quad + \alpha e\phi^{-3}\tilde{T}^{\lambda\rho ab}[-3\mathcal{I}_\lambda(\Omega_-)\delta\Omega_{\rho-}^{ab} + \frac{9}{2}\sqrt{2}H^\nu{}_{\lambda\rho}\delta\Omega_{\nu-}^{ab} - \frac{1}{2}\delta\bar{\psi}^\nu{}_\lambda\Gamma_{\rho ab}\psi_\nu - \frac{1}{4}\delta\bar{\psi}_{\lambda\rho}\Gamma^\mu\Gamma_{ab}(\psi_\mu + \frac{1}{3}\sqrt{2}\Gamma_\mu\lambda)], \quad (12)
\end{aligned}$$

where

$$\tilde{T}_{\mu\nu\rho\sigma} = -2\sqrt{2}\partial_{[\mu}H_{\nu\rho\sigma]} = -\frac{1}{12 \times 6!}i\sqrt{2}\epsilon_{\mu\nu\rho\sigma\alpha_1\dots\alpha_6}\mathcal{A}^{\alpha_1\dots\alpha_6}, \quad (13)$$

with $\mathcal{A}^{\alpha_1\dots\alpha_6}$ the equation of motion of $A_{\alpha_1\dots\alpha_6}$.

Using the Lemma, we can therefore cancel the variation (10) in a different way. In that case the $O(\alpha)$ transformation rules of the supergravity fields are

$$\begin{aligned}\delta_\alpha e_\mu{}^a &= 2\alpha e^{-1} \phi^3 g_{\mu\rho} \mathcal{L}_\lambda(\Omega_+) (e\phi^{-3} e^{\nu a} \delta_{\text{extra}} \Omega_{\nu-}^{\lambda\rho}) + \alpha (\delta_{\text{extra}} \bar{\psi}_{\lambda\mu}) \Gamma^a \psi^\lambda, \\ \delta_\alpha \phi &= 3\phi e^{\mu a} \delta_\alpha e_{\mu a}, \quad \delta_\alpha \psi_\mu = 2\alpha e^{-1} \phi^3 g_{\mu\rho} \mathcal{L}_\lambda(\Omega_+) (e\phi^{-3} \delta_{\text{extra}} \psi^{\lambda\rho}), \quad \delta_\alpha \lambda = -\frac{1}{4} \sqrt{2} \Gamma^\mu \delta_\alpha \psi_\mu, \\ \delta_\alpha A_{\alpha_1 \dots \alpha_6} &= \frac{1}{12 \times 6!} i \sqrt{2} \alpha e \phi^{-3} \epsilon^{\lambda\rho ab}{}_{\alpha_1 \dots \alpha_6} [-3 \mathcal{L}_\lambda(\Omega_-) \delta_{\text{extra}} \Omega_{\rho-}^{ab} + \frac{9}{2} \sqrt{2} H^\nu{}_{\lambda\rho} \delta_{\text{extra}} \Omega_{\nu-}^{ab} \\ &\quad - \frac{1}{2} (\delta_{\text{extra}} \bar{\psi}^\nu{}_\lambda) \Gamma_{\rho ab} \psi_\nu - \frac{3}{4} (\delta_{\text{extra}} \bar{\psi}_{\lambda\rho}) \Gamma^\mu \Gamma_{ab} (\psi_\mu + \frac{1}{3} \sqrt{2} \Gamma_\mu \lambda)] .\end{aligned}\quad (14)$$

This ends the construction of the quadratic effective action. The fact that different sets of transformation rules leave the same action invariant to this order, is due to the iterative invariance which we employ here. The two versions are related by a redefinition of the supergravity fields.

We now proceed with the construction of the cubic effective action. Clearly (9), with the new transformation rules (11) or (14) at $O(\alpha)$, is not invariant to $O(\alpha^2, \alpha\beta)$. In the variation of the action of $O(\alpha^2, \alpha\beta)$ we have contributions from

(a) the $O(\alpha)$ transformations of the supergravity fields, applied to the *explicit* dependence of (2) and (9) on the supergravity fields.

(b) the $O(\alpha)$ transformations of $\Omega_{\mu-}^{ab}$ and ψ^{ab} induced by the $O(\alpha)$ -variations of the supergravity fields, and applied to the action (9).

The two sources (a) and (b) are treated separately for the following reason. Those of type (b) are easy to deal with. They can be cancelled by using the Lemma. Furthermore, since the $O(\alpha)$ variations of the supergravity fields, (11) or (14), are proportional to $O(\alpha^0)$ equations of motion, these variations of (9) are still quadratic in equations of motion. They therefore give rise to additional transformation rules of the supergravity fields, now of $O(\alpha^2)$, which are again proportional to equations of motion. Type (a), however, gives rise to a variation of the action that is linear in equations of motion. Therefore this contribution can also be cancelled, but now the resulting transformations will no longer contain equations of motion. We conclude that no $O(\alpha^2, \alpha\beta)$ action is required. When we proceed to the quartic action the $O(\alpha^2, \alpha\beta)$ transformation rules proportional to equations of motion are harmless, since they again give variations that can be cancelled trivially. The $O(\alpha^2, \alpha\beta)$ transformation rules which do not contain equations of motion determine the quartic action.

Finally we discuss the form of the $O(\alpha^3, \alpha^2\beta, \alpha\beta^2)$ quartic action. As explained above, this requires the variation of (2) and (9) due to the explicitly occurring supergravity fields, and the resulting $O(\alpha^2, \alpha\beta)$ transformation rules of the supergravity fields required to cancel this variation. Under an arbitrary variation of the explicit supergravity fields in (2) and (9) we find

$$\begin{aligned}(\delta_{\alpha^2} + \delta_{\alpha\beta})(\mathcal{L}_\alpha + \mathcal{L}_\beta) &= \delta_\alpha (e\phi^{-3}) e^{-1} \phi^3 (\mathcal{L}_\alpha + \mathcal{L}_\beta) - e\phi^{-3} \delta_\alpha e_\mu{}^a T^{\mu a} - \frac{1}{4} e\phi^{-3} \bar{X}_{ab} \Gamma_\mu \Gamma^{ab} (\delta_\alpha \psi_\mu + \frac{1}{3} \sqrt{2} \Gamma_\mu \delta_\alpha \lambda) \\ &\quad - \frac{1}{4} i \sqrt{2} \epsilon^{\alpha_1 \dots \alpha_6 \beta_1 \dots \beta_4} \delta_\alpha A_{\alpha_1 \dots \alpha_6} T_{\beta_1 \dots \beta_4} .\end{aligned}\quad (15)$$

In (15), and also in the subsequent results, we use the following tensors and tensor-spinors:

$$\begin{aligned}T_{\mu\nu\lambda\rho} &= \alpha R_{[\mu\nu}{}^{ab}(\Omega_-) R_{\lambda\rho]}{}^{ab}(\Omega_-) + \beta \text{tr} F_{[\mu\nu} F_{\lambda\rho]}, \quad T_{\mu\nu} = \alpha R_{\mu\lambda}{}^{ab}(\Omega_-) R^\lambda{}_\nu{}^{ab}(\Omega_-) + \beta \text{tr} F_{\mu\lambda} F^\lambda{}_\nu, \quad T = T_\mu{}^\mu, \\ X_{\mu\nu} &= \alpha R_{\mu\nu}{}^{ab}(\Omega_-) \psi^{ab} + \beta \text{tr} F_{\mu\nu} \chi, \quad X_\mu = \Gamma^\nu X_{\mu\nu}, \quad X = \Gamma^\mu X_\mu.\end{aligned}\quad (16)$$

To determine the $O(\alpha^2, \alpha\beta)$ transformation rules of the supergravity fields (type (a) mentioned above) we have to make a choice between the two alternative $O(\alpha)$ transformation rules (11) and (14). It turns out that the two possibilities lead to the same result for the quartic action. Let us choose (14).

So we substitute (14) in (15), while taking for δ_{extra} the equation of motion parts of the $\Omega_{\mu-}^{ab}$ and ψ^{ab} trans-

formation rules (6), (7). The result is then cancelled by assigning the following $O(\alpha^2, \alpha\beta)$ transformation rules to the supergravity fields:

$$\begin{aligned}(\delta_{\alpha^2} + \delta_{\alpha\beta})\psi_\mu &= \frac{9}{4}\alpha\sqrt{2}H_{[\lambda\mu}{}^a T_{\rho]a}\Gamma^{\lambda\rho}\epsilon + \frac{9}{4}\alpha e^{-1}\phi^3 e_\mu^{[a}\Gamma^{bc]}\epsilon(\mathcal{L}_\lambda(\Omega_-)(e\phi^{-3}T^{\lambda\rho ab}))e_\rho{}^c + \frac{27}{8}\alpha\sqrt{2}e_\mu^{[a}\Gamma^{bc]}\epsilon T_{\lambda\rho ab}H_c{}^{\lambda\rho}, \\(\delta_{\alpha^2} + \delta_{\alpha\beta})\lambda &= -\frac{1}{6}\sqrt{2}\Gamma^\mu(\delta_{\alpha^2} + \delta_{\alpha\beta})\psi_\mu, \\(\delta_{\alpha^2} + \delta_{\alpha\beta})A_{\alpha_1\dots\alpha_6} &= -\frac{1}{8\times 6!}\alpha\sqrt{2}e\phi^{-3}\epsilon\Gamma^{\lambda\rho}_{\alpha_1\dots\alpha_6}[\Gamma^a\psi_\lambda T_{\rho a} + 2\mathcal{L}_\lambda(\Omega_+)X_\rho + \frac{1}{2}\Gamma^{\mu ab}\psi_\lambda T_{\rho\mu ab} \\&\quad + \frac{3}{4}\Gamma^\mu\Gamma^{ab}(\psi_\mu + \frac{1}{3}\sqrt{2}\Gamma_\mu\lambda)T_{\lambda\rho ab}].\end{aligned}\quad (17)$$

We are now ready to calculate the variation of the action at $O(\alpha^3, \alpha^2\beta, \alpha\beta^2)$. The only contribution to this variation which we cannot cancel trivially is due to the transformations (17), applied to the explicitly occurring supergravity fields in (2) and (9). These are the only variations of the action at this stage that are not proportional to the equations of motion of the supergravity fields. So we substitute (17) in (15), to obtain

$$\begin{aligned}(\delta_{\alpha^3} + \delta_{\alpha^2\beta} + \delta_{\alpha\beta^2})\mathcal{L} &= \frac{9}{16}\alpha\bar{X}_{ab}(-\Gamma_f\Gamma^{ab}\Gamma_{cd} + \frac{2}{3}\Gamma^{ab}\Gamma_f\Gamma_{cd})\epsilon(\mathcal{L}_\lambda(\Omega_-)(e\phi^{-3}\Gamma^{\lambda\rho cd}))e_\rho{}^f \\&\quad + \frac{3}{4}\alpha e\phi^{-3}T^{\lambda\rho cd}\bar{\epsilon}\Gamma_{cd}[\Gamma^a\psi_\lambda T_{\rho a} + 2\mathcal{L}_\lambda(\Omega_+)X_\rho + \frac{1}{2}\Gamma^{\mu ab}\psi_\lambda T_{\rho\mu ab} + \frac{3}{4}\Gamma^\mu\Gamma^{ab}(\psi_\mu + \frac{1}{3}\sqrt{2}\Gamma_\mu\lambda)T_{\lambda\rho ab}].\end{aligned}\quad (18)$$

This variation has to be cancelled by the introduction of additional $O(\alpha^3, \alpha^2\beta, \alpha\beta^2)$ terms, i.e., an action quartic in the Riemann tensor is required.

Fortunately there is no need to perform again the Noether procedure to achieve this cancellation. At this point we can use the result of our previous work [1] with the two-index formulation. In that case we had a similar $O(\alpha^3, \alpha^2\beta, \alpha\beta^2)$ variation of the action (eq. (3.22) of ref. [1]), and to cancel it we introduced a general quartic action with a priori arbitrary coefficients (eq. (4.6) of ref. [1]). To determine these coefficients, it was sufficient to consider variations of the type $\epsilon\lambda T^2$, $\partial T\epsilon X$ and $T\epsilon\partial X$. The bosonic part of the quartic action is completely determined by the $\epsilon\lambda T^2$ cancellation. Comparing (18) to (3.22) of ref. [1], we see that the $\epsilon\lambda T^2$ terms to be cancelled are exactly equal! This implies that the bosonic quartic action in the dual formulation has the same functional form as in the two-index version. We should recall, of course, the difference in the interpretation of H , which in both cases occurs as torsion in the quartic action.

The $\partial T\epsilon X$ and $T\epsilon\partial X$ terms in (10) are not exactly equal to those in (3.22) of ref. [1]. However, a comparison with the calculation done in ref. [1] shows that only two of the fermionic contributions to the quartic action have to be modified. To be precise, the terms

$$+b_2\partial_\mu T^{\mu\nu\lambda\rho}(\alpha\bar{\psi}^{ab}\Gamma_{\lambda\nu\rho}\psi_{ab} + \beta\text{tr}\bar{\chi}\Gamma_{\lambda\nu\rho}\chi) + c_4 T^{\mu\nu\lambda\rho}\bar{\psi}_\mu\Gamma_{\nu\lambda}X_\rho\quad (19)$$

of (4.6), which had $b_2 = -\frac{1}{8}$ and $c_4 = -\frac{3}{4}$ in ref. [1], now require

$$b_2 = -\frac{1}{2}, \quad c_4 = +\frac{3}{4}.\quad (20)$$

As a check we have verified the cancellation of variations of the type $\epsilon\psi_\mu T^2$.

Note that the $O(\alpha^3, \alpha^2\beta, \alpha\beta^2)$ variation of the action vanishes modulo a number of contributions which are again proportional to equations of motion of the supergravity fields, and now also of the Yang-Mills fields (since they always occur together in the combinations X and T , or as in the first term in (19)). The terms in (19) play no role in these equation of motion terms and in the corresponding new transformation rules. Therefore these remain as in ref. [1].

This concludes the construction of the quartic action. For convenience we summarize the main result at this point. The bosonic effective action reads

$$\begin{aligned}\mathcal{L} &= e\phi^{-3}[-\frac{1}{2}R(\omega(e)) + \frac{3}{4}H_{\mu\nu\rho}H^{\mu\nu\rho} + \frac{9}{2}(\phi^{-1}\partial_\mu\phi)^2 + \frac{1}{4}T - \frac{1}{4}i\sqrt{2}e^{-1}\phi^{-3}\epsilon^{\alpha_1\dots\alpha_6\beta_1\dots\beta_4}A_{\alpha_1\dots\alpha_6}T_{\beta_1\dots\beta_4} \\&\quad + \frac{3}{2}\alpha T^{\mu\nu\lambda\rho}T^{\mu\nu\lambda\rho} + \frac{1}{2}\alpha T^{\mu\nu}T^{\mu\nu}].\end{aligned}\quad (21)$$

The tensors T are defined in (16). The action is invariant under supersymmetry, with transformation rules given in (3) (δ_0), (14) (δ_α), (17) (δ_{α^2} , note that δ_α of the Yang–Mills fields vanishes), and contributions of $O(\alpha^3, \alpha^2\beta, \alpha\beta^2)$. Some of these arise from the cancellation of variations of \mathcal{L}_α , and can easily be derived using the formulae presented in this paper. There are also contributions from the Noether procedure at $O(\alpha^3)$ [1], which we now present for the fermions only:

$$\begin{aligned}
 (\delta_{\alpha^3} + \delta_{\alpha^2\beta})\psi_\mu &= 2\alpha^2 e^{-1} \phi^3 g_{\mu\rho} \mathcal{L}_\lambda(\Omega_+) \{ e\phi^{-3} [\frac{1}{32} T\Gamma^{\sigma\tau} R_{\sigma\tau}{}^{\lambda\rho}(\Omega_-) \\
 &\quad - \frac{1}{2} T^{\sigma\tau} \Gamma^\nu \Gamma_\sigma R_{\tau\nu}{}^{\lambda\rho}(\Omega_-) + \frac{1}{64} \Gamma^{\sigma\tau} \Gamma^{\alpha\beta\gamma\delta} T_{\alpha\beta\gamma\delta} R_{\sigma\tau}{}^{\lambda\rho}(\Omega_-)] \} \epsilon, \\
 (\delta_{\alpha^3} + \delta_{\alpha^2\beta})\lambda &= -\frac{1}{4} \sqrt{2} \Gamma^\mu (\delta_{\alpha^3} + \delta_{\alpha^2\beta}) \psi_\mu, \\
 (\delta_{\alpha^2} + \delta_{\alpha\beta})\chi &= -\alpha (\frac{1}{32} T\Gamma^{\sigma\tau} F_{\sigma\tau} - \frac{1}{2} T^{\sigma\tau} \Gamma^\nu \Gamma_\sigma F_{\tau\nu} + \frac{1}{64} \Gamma^{\sigma\tau} \Gamma^{\alpha\beta\gamma\delta} T_{\alpha\beta\gamma\delta} F_{\sigma\tau}) \epsilon.
 \end{aligned} \tag{22}$$

Invariance of the action requires other transformations of the fermions as well, but these are all proportional to equations of motion.

Let us now discuss the relation between the present results for the quartic effective action, and the results of ref. [1] in the two-form formulation (see Appendix A of ref. [1]). In ref. [1] $H_{\mu\nu\rho}$ is the field strength of the two-index gauge field, modified with Yang–Mills and Lorentz Chern–Simons forms, while in the dual formulation $H_{\mu\nu\rho}$ is the dual of the field strength of $A_{\alpha_1\dots\alpha_6}$ (4). The role the Chern–Simons terms is taken over by the ϵAT term in (22). We should stress that in the fermionic sector there are additional differences. In $\mathcal{L}_0 + \mathcal{L}_\alpha + \mathcal{L}_\beta$ there is a difference in the terms quartic in the fermions, while in the quartic action terms bilinear in fermions are different (19), (20). It will be interesting to see whether or not in higher orders also the bosonic part of the action may differ between the two formulations.

The differences discussed above can be understood in a natural way by considering a duality transformation between the two formulations which includes the quartic action. This will be discussed in a separate paper (see ref. [8], see also Bellucci et al. [4]).

The action $\mathcal{L}_\beta(F^2)$ (2) has an additional superconformal invariance, if the supergravity fields are interpreted as the fields of the $d=10$ superconformal Weyl multiplet [7]. For this superconformal aspect, the dual formulation is essential. One expects that conformal supergravity allows the existence of an R^2 invariant, where $R_{\mu\nu}{}^{ab}$ is now the Weyl tensor (a linearized version of this invariant was given in ref. [7]). It would be interesting if the R^2 -action (9) also admits a superconformal interpretation.

Finally, one may consider the possible compactification scenarios in the dual formulation. In this context, we note that the quartic effective action requires an additional $O(\alpha^3)$ term, which has no Yang–Mills analogue [10]. As in the two-index formulation, the fact that the symmetry between the Yang–Mills sectors disappears may have interesting consequences for compactification to $d=4$ and residual supersymmetry [11].

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